CERTIFIED PUBLIC ACCOUNTANT

FOUNDATION LEVEL 1 EXAMINATIONS

F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS

DATE: THURSDAY, 30 NOVEMBER 2023

MARKING GUIDE AND MODEL ANSWERS

INSTRUCTIONS:

1. **Time Allowed: 3 hours 15 minutes** (15 minutes reading and 3 hours writing).
2. This examination has **seven** questions and only **five** questions **should** be attempted.
3. Marks allocated to each question are shown at the end of the question.
4. Show all your workings where applicable.
5. The question paper should not be taken out of the examination room.

QUESTION ONE

Marking Guide

Criteria	**Marks**
a. Formula of netbook value for HOWO, CHACMAN and ISUZU	3
The formula for disposal gain/loss	2
Disposal gain or loss figure for each truck)	3
Total disposal gain or loss figure for all trucks	2
b. Formulation of the system of equations (1 mark per each equation)	3
Formulation of revenue function	3
Total revenue for serving 20 depots (2 marks for substitution,2 marks for correct figure)	4
Total	**20**

Model Answers

a) Disposal gain or loss for truck HOWO:

HOWO truck was acquired three years ago for 16,800,000 FRW and has a net book value of 7,200,000 Frw. A revenue of 22,000,000 FRW was received by disposing off this truck.

By substituting the above values in the equation V=a-bt we get;

where V is net book value of asset
b is annual depreciation
t is time
So V =16,800,000- bt
For t = 3 years (given) = 7,200,000=16800,000-bt= 7,200,000=16,800,000-3b
= 7,200,000-16,800,000= -3b
-9,600,000=-3b
b = -9,600,000/-3 = 3200,000

The truck was disposed-off after five years of service, so, the net book value will be calculated as follows: V=16,800,000-3,200,000(5) V= FRW 800,000

Disposal gain/(loss)-HOWO Truck=Sales revenue-Net book value
=22,000,000-800,000
= **FRW 21,200,000 (Gain)**

Disposal gain or loss for truck CHACMAN:

The truck was acquired four years ago for FRW 33,600,000 and has current a net book value of FRW 20,160,000. A revenue of FRW 11,400,000 was received after disposing off this truck.

V=a-bt 20,160,000=a-4b
For t = 4 years = 20,160,000=33,600,000 -4b =-13,440,000 = -4b
b= -13,440,000/-4

b=3,360,000

The truck was disposed-off after five years of service, so, the net book value will be calculated as follows V=33,600,000-3,360,000t Net book value after 5 years=33,600,000- (3,360,000*5) V=16,800,000

Disposal gain/(loss)-CHACMAN Truck= Sales revenue-Net book value

= 11,400,000-16,800,000

= FRW (5,400,000) (Loss)

Disposal gain or loss for truck ISUZU:

The truck was acquired two years ago and currently has an accumulated depreciation of FRW 13,440,000 while the acquisition cost was FRW 67,200,000, hence the net book value is FRW 53,760,000 (67,200,000-13,440,000).

V=a-bt 53,760,000=a-2b

If t=2 53,760,000=67,200,000-2b

=53,760,000-67200,000=-2b

b=-13,440,000/-2 =6,720,000

V=67,200,000-6,720,000t

Net book value after 5 years=67,200,000- (6,720,000*5) V=33,600,000

Disposal gain/(loss)-ISUZU Truck= Sales revenue-Net book value

= 19,000,000-33,600,000

=-FRW (14,600,000) (Loss)

Total disposal gain/(loss)= Disposal gain/(loss)-HOWO+ Disposal gain/(loss)-CHACMAN+ disposal gain/(loss)-ISUZU= 21,200,000-5,400,000-14,600,000

= FRW 1,200,000 (Gain)

b) The amount of revenue to be earned by serving 20 depots

Let **R**: be revenue from transport services **a, b** and **c**: constants **X:** number of depots served

By using the above data, let's substitute in the revenue function $R=ax^2+bx+c$ (rounded in Million)

$12,580=a(10)^2+b(10)+c$ $100a+10b+c=12,580$

$13,280=a(15)^2+b(15)+c$ $225a+15b+c=13,280$

$13380= a(30)^2+b(30)+c$ $900a+30b+c=13,380$

We can solve the above equations with gauss elimination method

$100a+10b+c=12,580$ *15

$225a+15b+c=13,280$ *(-10)

$1,500a+150b+15c=188,700$

$-2,250a-150b-10c=-132,800$

$225a+15b+c=13,280$ *2

$900a+30b+c=13,380$ *(-1)

$450a+30b+2c=26,560$

$-900a-30b-c=-13,380$

-750a+5c=55,900 (i) **-450a+c=13,180 (ii)**

-750a+5c=55,900 |*(1)
-450a+c=13,180 |*(-5)
-750a+5c=55,900
2250a-5c=-65,900
1500a=-10,000 a= (-10,000/1500) a= -20/3**

We can substitute to get the value of b and c as follows:

-750(-20/3)+5c=55,900 5000+5c=55,900 5c=50,900 c=10,180**
100a+10b+c=12,580 100(-20/3) +10b+10,180=12,580
-2000/3+10b+10,180=12,580 10b=12,580-10,180+2000/3 30b=7200+2000 b=920/3**

$R=-20/3\ x^2+ 920/3\ x+10180$
$R=((-20/3)*(20)^2)+((920/3)*20)+10180$
R=-8000/3+18,400/3+10180 **R= FRW 13,646,670,000**
Total revenue to be earned by serving 20 depots is FRW 13,646,670,000

QUESTION TWO

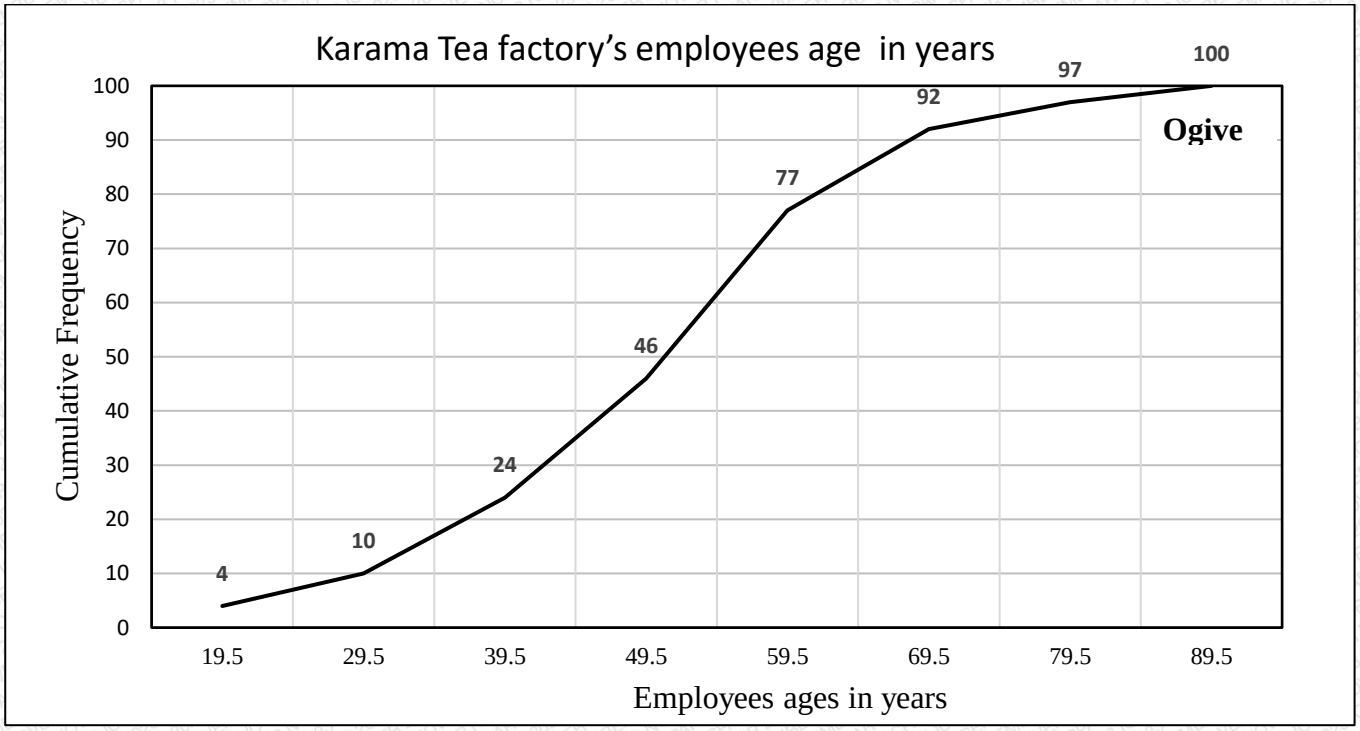
Marking Guide **Marks**

a. Correct formulation of class boundaries (0.5 mark per each class boundary) 4
Cumulative frequency 2
Ogive graph 4
b. Explanation of six uses, 1 mark each and four limitations of statistics,1 mark per each. 10
Total **20**

Model Answers

a. A smoothed ogive with the aid of data given.

Class	Frequency	Class boundaries	Upper Class Limits	Cumulative Frequency
10-19	4	9.5-19.5	19.5	4
20-29	6	19.5-29.5	29.5	10
30-39	14	29.5-39.5	39.5	24
40-49	22	39.5-49.5	49.5	46
50-59	31	49.5-59.5	59.5	77
60-69	15	59.5-69.5	69.5	92
70-79	5	69.5-79.5	79.5	97
80-89	3	79.5-89.5	89.5	100



b. Six uses and four limitations of statistics in business analysis

Statistics can be used for various functions but the most essential include:

- Presenting data in a definite form which makes the statement logical and convincing. Numerous figures can be summarized into a single intelligible figure.
- Statistics reduces the complexity of data. Usually, raw data is unintelligible. Ling different statistical measures such as averages, graphs, dispersions, etc.. The unintelligible data can be made simple and can easily be interpreted and conclusions drawn.
- **Facilitating comparison.** Comparison wouldn't be possible without the use of the various statistical measures. Both graphical and numerical measures provide ample scope for comparison.
- **Establishing trends and tendencies**. After studying data over a period of time, a trend can be established which can help in forecasting. Planners can forecast the future produce in agriculture, future population by considering present day figures.
- **Drawing valid conclusions/inferences**. Statistical measures can be used draw conclusions from a given survey which can be used to evaluate different projects.
- **Testing hypotheses.** Statistics can be used to test the truth of new ideas. This helps in developing new theories.

. **Limitations of statistics**

- The methods used in statistics only deal with quantitative attributes of data and leave out the qualitative ones, that is, the methods are not exhaustive. Quantitative attributes include figures and facts. They also answer questions, such as "how much", "how many", while qualitative attributes cannot be expressed numerically. and this includes integrity, honesty, intelligence and colour, the statistical methods used cannot bring out these characteristics, hence, a limitation to statistics.

- Statistical laws used only apply to the average of a given sample, but when it comes to only one point or an individual from the same sample, the parameter being measured may deviate from the average for the sample. These statistical laws apply to only specific conditions and, therefore, cannot be applied universally. These laws are true on average, for example, one may say that on average all students who sat for a certain paper passed it. Therefore, statistics sometimes gives results on averages and not being 100% reliable, hence, a limitation of statistics.

- Some statistical methods used cannot be applied to heterogeneous data, that is the sample is not uniform in all characteristics. For example, one may be interested in collecting data about the characteristics of a certain tribe from a particular community but some of the respondents may be living within the same community but from a different tribe and hence gives incorrect information.

- Although it is true for the average of the sample, the choice of the sample from the population as a whole may be biased. The information obtained from different samples within the same population may give different results, hence, a limitation of statistics.

- The information obtained using statistical data or methods should be interpreted, bearing in mind the characteristics of the sample from which the data was collected in the absence of this knowledge about the sample, the information is liable to misuse, that is wrong interpretations or conclusions may be made. It is, therefore, imperative that the statistics must be used by experts or statisticians only.

QUESTION THREE

Marking Guide

a.	Decision making criteria under uncertainty (2 marks per each correct explanation)	8
b.	Correct computation of expected demand	2
	Formula of expected revenue per each machine (2 marks each)	6
	Expected net revenue of each machine (1 mark per each correct value)	3
	A correct recommendation of the best machine	1
	Total	**20**

Model Answers

a. The following are decision making criteria/methods under uncertainty:

1. Maximin criterion

In this decision criterion, for each action one determines the worst outcome (smallest reward) and chooses the best of the worst outcome.

2. Maxmax criterion

In this decision criterion, for each action one determines the best outcome (largest reward) and chooses the best of the best outcomes.

3. Minimax regret criterion

In this decision criterion, one uses the concept of opportunity cost to arrive at a decision. This is where the decision that endeavors to minimize the maximum loss that would arise from the decision taken.

4. Expected value Criterion

With this decision criterion, one uses probability to choose an action or alternative that yields the largest expected value.

5. Hurwiz method

With this method, the decision maker takes into account maximum and minimum payoff for each alternative and assigns them weights according to his degree of optimism or pessimism. The alternative which maximizes the sum of these weighted payoffs is then selected.

6.La Place method

This method uses all the information by assigning equal probabilities to the possible payoffs for each action and then selecting that alternative which corresponds to the maximum.

b. Recommending a machine that should be purchased by Urwunge Winery (R) Ltd

Expected demand of Yuliana wine= (200*0.3)+(400*0.6)+(800*0.1)

=60+240+80

=380 Bottles

Expected revenue from Chinese Machine CMX002 R= 20x-(10+3) x+20-200 in thousands

R=20x-13x-180 in thousands **R=7x-180 in thousands**

By substituting the value of x=380 R= (7*380)-180 **R=2,480 FRW thousands**

Expected revenue from Italian Machine YB009 R= 20x-(10+2) x+40-400 in thousands

R=20x-12x-180 in thousands **R=8x-360 in thousands**

By substituting the value of x=380 R= (8*380)-360 **R=2,680 FRW thousands**

Expected revenue from Korean Machine KW978 R= 20x-(10+1) x+80-800 in thousands

R=20x-11x-720 in thousands **R=9x-720 in thousands**

By substituting the value of x=380 R= (9*380)-720 **R=2,700 FRW thousands**

It is recommended to acquire Korean Machine KW978 since it earns the company a highest net revenue of 2,700,000 FRW.

QUESTION FOUR
Marking Guide

Criteria	**Marks**
Assumptions of linear regression (1 mark each)	4
Correct difference between a type I error and a type II error (1 mark each)	2
Allocate one mark for each of these summations ($\sum x, \sum y, \sum x^2, \sum x^3, \sum x^4, \sum x^2y$)	6
The regression line equation	4
Forecasted value of fertilizer in 2024 and 2026 (0.5 marks each formular, workings 1 mark each and 1 mark for result.)	4
Total	**20**

Model Answers

a. Regression is a way of analyzing relationship between dependent variable (response) and one or more independent variables (predictors).

Assumptions of linear regression analysis:

- Linear regression analysis assumes linearity between variables
- All the variables are normally distributed
- The expected mean error of the regression model is zero
- The variance of the errors is constant (homoskedasticity)
- The errors are independent (no autocorrection)

b. Type I error refers to rejecting the null hypothesis when it is true. This occurs with probability α while Type II error refers to the failure to reject the null hypothesis when it is false; this occurs with probability β.

c. **Given the equation relating the fertilizer's cost and time:** $y=a_0+a_1x+a_2x^2$

$\sum y=a_0N+a_1\sum x+a_2\sum x^2$ $\sum xy=a_0\sum x+a_1\sum x^2+a_2\sum x^3$ $\sum x^2y=a_0\sum x^2+a_1\sum x^3+a_2\sum x^4$

$a =(\sum y-b\sum x)/n$ $b=((n\sum xy)- (\sum x\sum y))/((n\sum x^2)- (\sum x)^2)$

$y=a_0+a_1x+a_2x^2$

Year	**x**	**y**	**x^2**	**x^3**	**x^4**	**xy**	**x^2y**
2014	-4	36.4	16	-64	256	-145.6	582.4
2015	-3	52.8	9	-27	81	-158.4	475.2
2016	-2	69.6	4	-8	16	-139.2	278.4
2017	-1	89.6	1	-1	1	-89.6	89.6
2018	0	124	0	0	0	0	0
2019	1	148	1	1	1	148	148
2020	2	164	4	8	16	328	656
2021	3	223.4	9	27	81	670.2	2010.6
2022	4	257.6	16	64	256	1030.4	4121.6

	$\sum x=0$	$\sum y=1,165.4$	$\sum x^2=60$	$\sum x^3=0$	$\sum x^4=708$	$\sum xy=1,643.8$	$\sum x^2y=8,361.8$

$\sum y=a_0N+a_1\sum x+a_2\sum x^2$ $1,165.4= 9a_0+(0)a_1 +60\ a_2$ $9a_0+ +60\ a_{2=}1,165.4$ (i)

$\sum xy=a_0\sum x+a_1\sum x^2+a_2\sum x^3$ $1643.8=a_0(0)+60a_1 +a_2(0)$ $1643.8=60a_1$ **» a_1= 27.4**

$\sum x^2y=a_0\sum x^2+a_1\sum x^3+a_2\sum x^4$ $8,361.8=60a_0+a_1(0) +708a_2$ $60a_0+708a_2=8,361.8$ (ii)

The equations in (i) and (ii) have to be solved simultaneously as follows:

$9a_0+ +60\ a_{2=}1,165.4$ / * 20

$60a_0+708a_2=8,361.8$ *(-3)

$180a_0+ +1200\ a_{2=}23,308$

$-180a_0-2,124a_2=-25,085.4$

$-924\ a_2=-1,777.4$

$a_2=$ (1,777.4/924) **» a_2=1.92**

By substitution, the value of a_0 is calculated as follows:

$9a_0+ +60(1,777.4/924)\ _=1,165.4$ $9a_0=$ 1,165.4-115.4155 **»a_0=116.7**

The linear regression line **$y=116.7+27.4\ X+1.92\ x^2$**

Forecasted fertilizer's cost for the year of 2024 and 2026 is calculated as below:

Year	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	**2024**	2025	**2026**
X	-4	-3	-2	-1	0	1	2	3	4	5	**6**	7	**8**

In 2024: $y=116.7+27.4\ (6) +1.92(6)^2$ $y=116.7+164.4+69.12$ **y=350.22 FRW million**

In 2026: $y=116.7+27.4\ (8)+1.92(8)^2$ $y=116.7+219.2+122.88$ **y= 458.78 FRW million**

QUESTION FIVE

Marking Guide

Criteria	**Marks**
a. Six assumptions of linear programming (1 mark each)	6
Four application of linear programming (1 mark each)	4
b. Constraint equations and non-negativity (1 Mark each)	6
Objective equation of the linear programming model (4 Marks)	4
Total	**20**

Model Answers

a. **Assumptions of linear programming**

- **Certainty**: Values in the objective and constraint are known with certainty and do not change during the period being studied
- **Proportionality/Linearity**: It assumes proportionality in the objective function and constraints inequalities
- **Additivity**: The total of all activities is given by the sum total of each activity conducted separately
- **Divisibility/continuity:** Solutions are easily divisible
- **Non negativity/finite choice:** Negative values of physical quantities are impossible
- **Time factors are ignored:** All productions are assumed to be instantaneous
- Costs and benefits which cannot be quantified easily are ignored
- Interdependence between demand of products is ignored, products may be complementary or a substitute for one another

Application of linear programing:

- Determination of optimal product mix in industries
- Determination of optimal machine and labour contribution
- Determination of optimal use of storage and shipping facilities
- Determining the best route in transport industries
- Determining investment plans
- To find the appropriate number of financial auditors
- Assigning advertising expenditure to different media plans
- Determining the amount of fertilizer to apply per acre of land in agricultural sector
- Determining campaign strategies in politics
- Determining the best marketing strategies

b. **A linear programming model to maximize Nguvu's revenue**

Let the following products be represented as follows:

Floor tile in factory A be: X_1
Wall tile in factory A be: X_2
Floor tile in factory B be: X_3
Wall tile in factory A be: X_4

Objective function:

Maximize Z= (170-80)X1+(160-80)X2+(170-100)X3+(160-100)X4

$Z=90X_1+80X_2+70X_3+60X_4$

Subject to:

- $3X_1+X_2 \leq 12{,}000$ (Clay constraint at factory A)
- $2X_1+X_2 \leq 10$ (Kiln time constraint at factory A)
- $3X3+2X4 \leq 15{,}000$ (Clay constraint at factory B)
- $X_3+X_4 \leq 4$ (Kiln time constraint at factory B)
- $8X1+6X2+7X3+5X4 \leq 80{,}000$ (Gas constraint-all factories)
- Non negativity: $X_1, X_2, X_3, X_4 > 0$

Note: The main objective is to maximize net revenue (Revenue exclusive of transport cost)

QUESTION SIX
Marking Guide

Criteria	**Marks**
a. Two imitations of CPM and two for PERT (1.5 marks each)	6
b.i. A network diagram (0.5 Mark for each correct activity)	4.5
Calculation of expected time (0.5 marks for each correct value)	4.5
ii. The critical path 3 marks, and project completion time 2 Marks.	5
Total	**20**

Model Answers

(a) limitations of Critical Path Method (CPM) and project Evaluation and Review Technique (PERT)

Critical Path Method (CPM)

- CPM is based on the assumption of known time for each of the activity in the project which may not be true in real life.
- For determining the time estimates, CPM does not incorporate statistical analysis.
- When certain changes are introduced in the network, the entire evaluation of the project is to be repeated and a new critical path is found.

Project Evaluation and Review Technique (PERT)

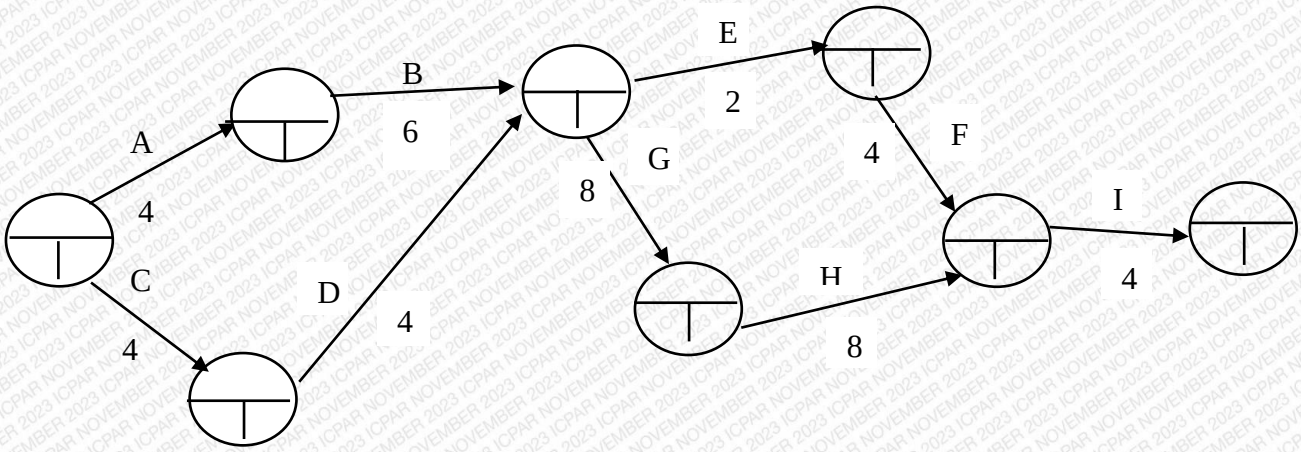
- PERT is based on the time estimates rather than known time for each activity.
- It emphasises only time and not costs.
- It is not practicable for routine planning of recurring activities.
- In PERT, the calculations based on probabilities are carried out on the assumption of independent activities. The distribution of total time is assumed to be normal but in real life this may not be true.
- For active control of a project, it requires frequent updating and revising of the PERT calculations

(b) (i) The expected time for each of the project's activities is calculated below:
Expected time=(O+P+4ML)/6
Where **O:** Optimistic time **P:** Pessimistic time **ML:** Most likely/probable time
Variance=$((p-o)/6)^2$ Remember that any number raised to the power of zero equals to one.

Activity	**Preceding activity**	**Time (weeks)**			**Expected time**	**Variance**
		Optimistic	**Most probable**	**Pessimistic**		
A	-	3	4	5	4	0.11
B	A	4	5	12	6	1.78

C	-	2	4	6	4	0.44
D	C	3	4	5	4	0.11
E	B,D	1	2	3	2	0.11
F	E	2	4	6	4	0.44
G	B,D	6	7	14	8	1.78
H	G	6	8	10	8	0.44
I	F,H	3	4	5	4	0.11
					44.00	**5.33**

A network diagram of the project's activities



Possible paths:

ABEFI=20 Weeks (4+6+2+4+4)
ABGHI=30 Weeks (4+6+8+8+4) **
CDEFI=18 weeks (4+4+2+4+4)
CDGHI=28 weeks (4+4+8+8+4)
Our critical path is made of activities ABGHI and it has a duration of 30 weeks

ii. The expected project completion time corresponds to the duration of the critical path which is 30 weeks

ABGHI=30 Weeks (4+6+8+8+4) **

QUESTION SEVEN

Marking Guide

Criteria	**Marks**
a. Explanation of components of time series (2 marks each)	6
b. For (i),(ii),(iii),(iv) evaluate a formula and correct answer(1 mark each)	8
For (v)and (vi) for correct formula (1.5 marks) for correct answer (1.5 marks)	6
Total	**20**

Model Answers

(a) Components of time series

The main components of time series are:

i. Secular Trend

The general tendency of the time series to increase or decrease or stagnate during a long period of time is called secular trend or simply trend. This phenomenon is usually observed in most of the series that show growth. For example, an upward tendency is usually observed in time series relating to population, production and sales of products, prices, income, bank deposits, etc. while a downward tendency is noticed in the time series relating to death, epidemics etc. due to advancement in technology, improved medical facilities etc. Secular trend is regular, smooth and long-term movement of a statistical series. It reveals the general tendency of the data.

ii. Season variation

It represents a periodic movement where the period is no longer than one year. The factors which mainly cause this type of variation in time series are climatic changes of the different seasons and the customs and habits which people follow at different times. The short-range stock and brisk periods of business activity at different seasons of the year, production and consumption of commodities, sales and profits of a company, etc. are in fact attributed to seasonal variations. The main objective of the measurement of seasonal variations is to isolate them from the attend and study their effects. A study of the seasonal patterns is extremely useful to businessmen, producers, sales managers etc. It helps in planning future operations and formulation of policy, in decisions regarding purchase or production, inventory control, personal requirements, selling and advertising.

iii. Cyclical variations

Cyclical variations or fluctuations are another type of periodic movement, with a period more than one year. Such movements are fairly regular in nature. One complains period is called a cycle. Cyclical fluctuations are found to exist in most of the business and economic time series.

iv. Irregular variation

Irregular variations or 'movements are -such variations which are completely unpredictable in character. These are unforeseen variations usually caused by factors which are either wholly unaccountable or caused by such unforeseen events as war, flood strikes and lockouts etc. These may sometimes be the result of many small forces each of which has a negligible effect but their combined effect is not negligible. They are in most cases beyond human control.

b. (i) The probability that only one fan is operating

P(A)=0.3 P($\overline{A}$)=0.7 P(B)=0.25 P($\overline{B}$)=0.75 P(C)=0.35 P($\overline{C}$)= 0.65

For each of the probability of operating, we need to calculate the probability of not operating as denoted by the line on each concerned fan above.

The probability that only one transformer is working will be calculated as follows:

A*$\overline{B}$*$\overline{C}$= 0.3*0.75*0.65	A*$\overline{B}$*$\overline{C}$= 0.146
$\overline{A}$*B*$\overline{C}$=0.7*0.25*0.65	$\overline{A}$*B*$\overline{C}$=0.113
$\overline{A}$*$\overline{B}$*C=0.7*0.75*0.35	$\overline{A}$*$\overline{B}$*C=0.183
	Total=0.442

The probability of finding one fan operating is 0.442 or 44.2%

(ii) The probability of finding Two fans operating is calculated below:

A*B*$\overline{C}$= 0.3*0.25*0.65	A*B*$\overline{C}$= 0.0487
A*$\overline{B}$*C=0.3*0.75*0.35	A*$\overline{B}$*C=0.078
$\overline{A}$*B*C=0.7*0.25*0.35	$\overline{A}$*B*C=0.061
	Total= 0.187

The probability of finding two fans operating is 0.187 or 18.7%

(iii) The probability of finding all three fans operating:

P(A)=0.3 P($\overline{A}$)=0.7 P(B)=0.25 P($\overline{B}$)=0.75 P(C)=0.35 P($\overline{C}$)= 0.65

The probability of finding all three fans operating =P(A)*P(B)*P(C)=0.3*0.25*0.35

=0.3*0.25*0.35

=0.026

(iv) The probability that None of the fans is operating:

P($\overline{A}$)*P($\overline{B}$)*P($\overline{C}$)= 0.7*0.75*0.65

=**0.341**

(v) The probability that at least 2 fans are operating=P(exact two working)+P(all three working)= 0.187 +0.026

=0.213

(vi) The probability that at most 2 fans are operating =P (none working) +P (one working) +P (two working) =0.341+0.442+0.187

=0.97 or 97%

END OF MARKING GUIDE AND MODEL ANSWERS